Ch. 9 - Center of Mass & Momentum

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Main Point of Ch. 9 :

Momentum Conservation! $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

For a system with translational invariance,

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$$

$$\star \quad m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} \quad \star$$

Types of Collisions

1. inelastic collision most generic type; some energy lost

   perfectly inelastic collision collide and *stick*,

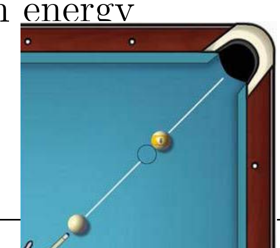
   $$\therefore \quad \vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_f$$

   $$m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = (m_1 + m_2)\vec{\mathbf{v}}_f$$

2. elastic collision momentum *and* energy are conserved; e.g., medium energy billiards

   $$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$
   $$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

Active Learning Exercise

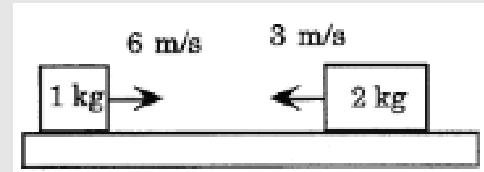
Problem: Perfectly Inelastic Collision

Two boxes are on a horizontal, frictionless surface. The boxes are sliding toward one another and subsequently collide. If the boxes stick together, **perfectly** *in***elastically**, what velocity has the 1 kg box after the collision?

Active Learning Exercise

Problem: Elastic Collision

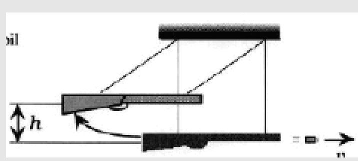
Two boxes are on a horizontal, frictionless surface. The boxes are sliding toward one another and collide. If the collision is **elastic**, meaning energy is conserved, what velocity has the 1 kg box after the collision? [use *solve*]



Active Learning Exercise

Problem: Example:

A 6.0 kg rifle is suspended by two long strings as shown. The rifle is fired and the recoil results in the rifle swinging backward to a maximum height $h = 0.040\,\text{m}$. What is the speed of the 0.010 kg bullet?



9.8 – 2D Collisions

- 2D Collisions

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$
$$m_1\,\vec{\mathbf{v}}_{1i} + m_2\,\vec{\mathbf{v}}_{2i} = m_1\,\vec{\mathbf{v}}_{1f} + m_2\,\vec{\mathbf{v}}_{2f}$$

- Or in x, y-component form:

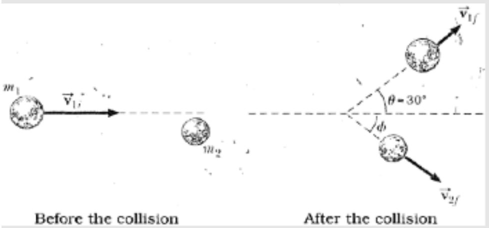
$$m_1\,v_{1i}\,\cos\theta_{1i} + m_2\,v_{2i}\,\cos\theta_{2i} = m_1\,v_{1f}\,\cos\theta_{1f} + m_2\,v_{2f}\,\cos\theta_{2f}$$

$$m_1\,v_{1i}\,\sin\theta_{1i} + m_2\,v_{2i}\,\sin\theta_{2i} = m_1\,v_{1f}\,\sin\theta_{1f} + m_2\,v_{2f}\,\sin\theta_{2f}$$

Active Learning Exercise

Problem: 2D *In*elastic w/ angles

A bowling ball with mass $m_1 = 2.0\,\text{kg}$ moving in the positive x direction with velocity $3.0\,\text{m/s}$ strikes a pin with mass $m_2 = 1.0\,\text{kg}$. The pin is initially at rest. Immediately after the collision, the bowling ball has a velocity of $1.5\,\text{m/s}$ at an angle of $30°$ with respect to the x-axis. What is the y-velocity of *the pin* after collision?



9.4 – Impulse

$$\vec{\mathbf{J}} = \int \vec{\mathbf{Force}} \cdot \mathrm{d}t$$
$$= \vec{\mathbf{Force}} \cdot \text{time}$$

9.4 – Momentum–Impulse Theorem

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o$$

- *Always* valid, if impulse takes into account all forces
- *Useful way* to calculate *change in direction*
- $\vec{\mathbf{J}}$ refers to the impulse delivered *to* an object

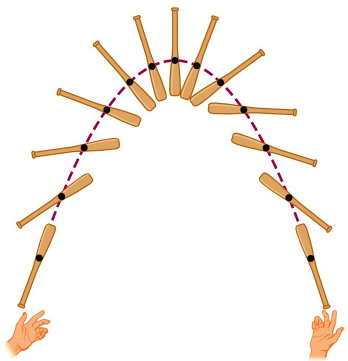
Active Learning Exercise

Problem: Bounces, Recoils & Rebounds

A $4.0\,\text{kg}$ zombie head is moving horizontally with a speed of $5.0\,\text{m/s}$ when it strikes a vertical wall. The head rebounds with a speed of $3.0\,\text{m/s}$. What is the magnitude of the impulse delivered to the head?

9.1 – Center of Mass

- momentum is thought of as centered on a *point*
- this point may be on *or near* the object
- all other points rotate about the **center of mass**



9.1 – Center of Mass

- For n discrete particles the c.o.m. is located at

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$

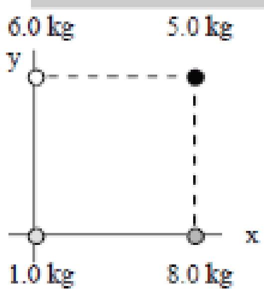
$$y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i = \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots}$$

$$z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i = \frac{m_1 z_1 + m_2 z_2 + \dots}{m_1 + m_2 + \dots}$$

- Such series are called **weighted average**

Active Learning Exercise

Problem: Center of Mass

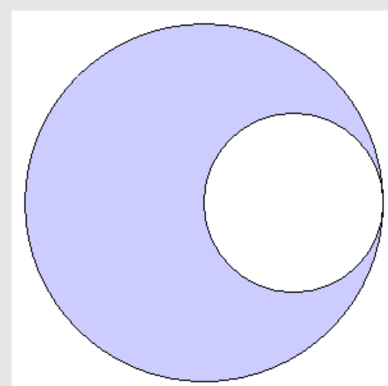


Four point masses are located at the corners of a square with sides 2.0 m, as shown. The 1.0 kg mass is at the origin. What is the vector location of the center of mass of the system?

9.1 – Center of Mass

Example:

Try this one!



Take $r = 1$, so $m_O = \pi$

$$x_{\text{com}} = \frac{\pi(0) + \frac{-\pi}{4}\left(\frac{1}{2}\right)}{\pi - \frac{\pi}{4}} = -\frac{1}{8}\frac{4}{3} = -\frac{1}{6}$$